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THERMOMECHANICAL PROCESSES IN THE FRICTION HEATING OF DISK BRAKES

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Thermomechanical processes are studied at the contact area of a metal brake disk and brake block during braking. Expressions are obtained for both the temperature and the thermal displacement in the center of the contact area caused by the effect of the friction heat source, whose power is a linear function of time.

A common disadvantage of mechanical friction brakes is the low wear resistance of the friction pairs, which depends on a number of factors, mainly temperature [1]. Therefore, during braking the temperature and the temperature-induced deformation of the surface at the friction contact area of a metal disk and blocks attract the close attention of designers and tribologists. During braking that is started at a high speed, the heat released has no time to heat the entire disk, as the process takes a short time. For this reason the temperature of its working surface is far in excess of its volume-average temperature. Moreover, in the intense brief braking the heat radiated into the environment can be neglected. Disks of alloy cast iron have the best wear resistance and strength [2]. The brake blocks are made of materials with low thermal conductivity; consequently, almost all the heat generated by friction is directed into the disk.

Because of the facts enumerated the problem of determining the heating temperature and thermal bulging of the surface of a brake disk can be formulated as a boundary-value problem of quasistatic thermoelasticity for the half-space in the circular surface region of which heat sources operate. In this case, Blok's solution [3] for a continuously generating heat source of a constant power is used as a basis. However, in the course of braking the amount heat generated by friction forces is a linear decreasing function of time [4]. In the literature there is no solution that would take into account this behavior of a friction source.

1. The temperature on the surface $z = 0$ of a half-space heated over a circular area $0 \le r \le a$ by a continuously generating heat source with power $q(r, \tau)$ is expressed as [5]

$$T(r, t) = \frac{1}{2K(\pi k)^{1/2}} \int_0^1 \int_0^a Q(s, \tau) \exp[-(R^2 + S^2)] I_0(2rS)\, s ds d\tau, \tag{1}$$

where

$$R^2 = \frac{r^2}{4k(t-\tau)}; \quad S^2 = \frac{s^2}{4k(t-\tau)}; \quad Q(r, \tau) = \frac{q(r, \tau)}{(t-\tau)^{3/2}}.$$

In braking the heat flux directed into the metal disk is [4]

$$q(r, t) = q_0 \left(1 - \frac{t}{t_s}\right) H(a - r) H(t_s - t). \tag{2}$$

A maximum of the axisymmetric temperature field (1) is at the point $r = 0$. Therefore, with (2) in view, we find from (1):

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$$T_{\max}(t) = T(0,t) = \frac{q_0}{2c_v k\sqrt{\pi k}} \int_0^t \int_0^a \frac{\exp(-S^2)}{(t-\tau)^{3/2}} \left(1 - \frac{\tau}{t_s}\right) s\,ds\,d\tau . \tag{3}$$

Integrating (3) with respect to s, we obtain

$$T_{\max}(t) = \frac{2q_0}{c_v\sqrt{\pi k}} \int_0^t \left(1 - \frac{\tau}{t_s}\right) \left\{1 - \exp\left[-\frac{a^2}{4k(t-\tau)}\right]\right\} (t-\tau)^{1/2}\,d\tau . \tag{4}$$

Letting

$$X = \frac{a^2}{4k(t-\tau)}, \tag{5}$$

$$\mathrm{Fo} = \frac{kt}{a^2}, \quad \mathrm{Fo}_s = \frac{kt_s}{a^2}, \tag{6}$$

formula (4) is rewritten as

$$T_{\max}(t) = \frac{q_0 a}{8c_v k^2\sqrt{\pi}} \int_{\frac{1}{4\mathrm{Fo}}}^{\infty} F(X)\,[1 - \exp(-X)]\,dX , \tag{7}$$

where

$$F(X) = \frac{4a^2X(\mathrm{Fo}_s - \mathrm{Fo}) + a^2}{t_s X^2\sqrt{X}} .$$

After calculation of the integrals on the right-hand side of (7), the following expression is obtained for the temperature in the center of the contact area

$$T_{\max}(t) = \frac{Q_0}{\sqrt{\pi}} \left\{\sqrt{4\mathrm{Fo}}\left(1 - \frac{2\mathrm{Fo}}{3\mathrm{Fo}_s}\right)\left[1 - \exp\left(-\frac{1}{4\mathrm{Fo}}\right)\right] + \right.$$

$$\left. + \frac{\sqrt{4\mathrm{Fo}}}{6\mathrm{Fo}_s}\exp\left(-\frac{1}{4\mathrm{Fo}}\right) + \left(1 - \frac{\mathrm{Fo}}{\mathrm{Fo}_s} - \frac{1}{6\mathrm{Fo}_s}\right)\sqrt{\pi}\,\mathrm{erfc}\,\frac{1}{2\sqrt{\mathrm{Fo}}}\right\}, \tag{8}$$

where $Q_0 = q_0 a/K$, $\mathrm{erfc}\,\zeta = 1 - \mathrm{erf}\,\zeta$.

At short times t ($\mathrm{Fo} \to 0$) the functions $\exp(-1/4\mathrm{Fo})$ and $\mathrm{erfc}\,[2\sqrt{\mathrm{Fo}}]^{-1}$ are decreasing and Eq. (8) takes a simpler form:

$$T_{\max}(t) = 2Q_0\sqrt{\left(\frac{\mathrm{Fo}}{\pi}\right)}\left(1 - \frac{2\mathrm{Fo}}{3\mathrm{Fo}_s}\right). \tag{9}$$

Asymptotic expression (9) was originally obtained in [6]. With the additional assumption of $t \ll t_s$, it follows from (9) that

$$T_{\max}(t) = 2Q_0\sqrt{\left(\frac{\mathrm{Fo}}{\pi}\right)} . \tag{10}$$

Relation (10) is a solution of the problem of a temperature "burst" on the half-space surface [7].

2. The normal displacement (thermal distortion) of the half-space surface $z = 0$ induced by its heating by heat flux (2) is equal to [8]:

$$u_z(r, t) = -\frac{\delta}{4\pi} \int_0^t \int_0^a \int_0^{2\pi} \frac{q(s, \tau)}{t - \tau} \Phi\left(\frac{3}{2}, 2; -Y\right) s ds d\theta d\tau , \tag{11}$$

where

$$\delta = \frac{\alpha_t (1 + \nu)}{K}; \quad Y = \frac{r^2 + s^2 - 2rs\cos\theta}{4k(t - \tau)} .$$

Proceeding from [9], the degenerate hypergeometric function $\Phi(3/2, 2; -Y)$ is expressed in the form

$$\Phi\left(\frac{3}{2}, 2; -Y\right) = \exp\left(\frac{Y}{2}\right)\left[I_0\left(\frac{Y}{2}\right) - I_1\left(\frac{Y}{2}\right)\right] . \tag{12}$$

With (2) and (12), the displacement in the center $r = 0$ of the heated circular region of the half-space surface is found from Eq. (11)

$$w_0(t) = -\frac{q_0 a^2 \delta}{8} \int_{\frac{1}{4\mathrm{Fo}}}^{\infty} F_1(X)\left[1 - \exp\left(-\frac{X}{2}\right) I_0\left(\frac{X}{2}\right)\right] dX , \tag{13}$$

where

$$w_0(t) = u_z(0, t); \quad F_1(X) = \left(1 - \frac{\mathrm{Fo}}{\mathrm{Fo}_s}\right)\frac{4}{X^2} + \frac{1}{X^3 \mathrm{Fo}_s};$$

X, Fo, and Fo_s are variables determined in accordance with formulas (5) and (6).

With the corresponding integrals in (13) calculated, the thermal displacement in the center of the contact area can be expressed as follows:

$$w_0(t) = B\left[-2\mathrm{Fo}\left(1 - \frac{\mathrm{Fo}}{2\mathrm{Fo}_s}\right) + \frac{1}{4}\left(1 - \frac{\mathrm{Fo}}{\mathrm{Fo}_s}\right)\Psi_1(\mathrm{Fo}) + \frac{1}{32\mathrm{Fo}_s}\Psi_2(\mathrm{Fo})\right] , \tag{14}$$

where

$$B = \delta K Q_0 a; \quad \Psi_m(\mathrm{Fo}) = \int_{\frac{1}{8\mathrm{Fo}}}^{\infty} \exp(-X) I_0(X) \frac{dX}{X^{m+1}} \quad (m = 1, 2) .$$

The functions $\Psi_m(\mathrm{Fo})$ are positive-definite. The maximum of the integrand function $\exp(-X) I_0(X)$ is 1 at $X = 0$. Consequently, $\Psi_m(\mathrm{Fo}) < \int_{1/8\mathrm{Fo}}^{\infty} dX/X^{m+1}$ and for $w_0(t)$ we have the estimate

$$\frac{w_0(t)}{B} \le \left[-2\mathrm{Fo}\left(1 - \frac{\mathrm{Fo}}{2\mathrm{Fo}_s}\right) + \frac{1}{4}\left(1 - \frac{\mathrm{Fo}}{\mathrm{Fo}_s}\right)\int_{\frac{1}{8\mathrm{Fo}}}^{\infty} \frac{dx}{x^2} + \frac{1}{32\mathrm{Fo}_s}\int_{\frac{1}{8\mathrm{Fo}}}^{\infty} \frac{dx}{x^3}\right] .$$

Hence, after calculation of the integrals, we have

$$u_z(0, t) \le 0 ,$$

that is, as a result of friction heating the disk surface is always convex.

At short times Fo $\to$ 0, Ψ_m(Fo) $\to$ 0, and

$$u_z(0, t) \cong -2B\text{Fo}\left(1 - \frac{\text{Fo}}{2\text{Fo}_s}\right). \tag{15}$$

Relation (15) gives a limit estimate of the thermal bulging of the working surface of the disk.

In an approximate calculation of the integrals Ψ_m(Fo) for arbitrary parameters Fo, we expand the modified Bessel function $I_0(x)$ into a power series in small values of the argument [10]

$$I_0(x) \cong \sum_{j=0}^{\infty} \frac{\left(\frac{x}{2}\right)^{2j}}{(j!)^2}, \quad x < x_0, \tag{16}$$

and asymptotics for large arguments [9]

$$I_0(x) \cong \frac{\exp(x)}{\sqrt{2\pi x}} \sum_{j=0}^{N} \frac{[(2j+1)!!]^2}{j!(8x)^j}, \quad x > x_0. \tag{17}$$

The point $x_0 \in R^+$ corresponds to the transition from formula (16) to (17) in calculating the modified Bessel function $I_0(x)$, and, according to [9], it is 3.75. The number N in (17) is chosen to achieve the desired calculation accuracy.

On the basis of (17) for Fo satisfying the inequality Fo $\leq 1/8x_0$, we obtain from (14):

$$W_0(\text{Fo}) = -2\text{Fo}\left(1 - \frac{\text{Fo}}{2\text{Fo}_s}\right) + g(\text{Fo}, A), \tag{18}$$

where

$$g(\text{Fo}, A) = \frac{8\text{Fo}\sqrt{\text{Fo}}}{\sqrt{\pi}} \sum_{j=0}^{N} \frac{[(2j+1)!!]^2}{j!} \left(\frac{A}{2j+3} + \frac{\text{Fo}}{\text{Fo}_s}\frac{1}{2j+5}\right)(\text{Fo})^j;$$

$$A = 1 - \frac{\text{Fo}}{\text{Fo}_s}; \quad W_0(\text{Fo}) = \frac{w_0(t)}{B}.$$

At Fo $> 1/8x_0$, from (16) we have the thermal distortion in the form

$$W_0(\text{Fo}) = -2\text{Fo}\left(1 - \frac{\text{Fo}}{2\text{Fo}_s}\right) + G(x_0, \text{Fo}) + g\left(\frac{1}{8x_0}, A\right). \tag{19}$$

Here,

$$G(x_0, \text{Fo}) = d_0(x_0, \text{Fo}) - \sum_{j=1}^{\infty} \frac{1}{2^{2j}(j!)^2} H_j(x_0, \text{Fo});$$

$$d_0(x_0, \text{Fo}) = \frac{A}{4}\left[d_1(x_0) - d_1\left(\frac{1}{8\text{Fo}}\right)\right] + \frac{1}{32\text{Fo}_s}\left[d_2(x_0) - d_2\left(\frac{1}{8\text{Fo}}\right)\right];$$

$$H_j(x_0, \text{Fo}) = \frac{A}{4}\left[D_{2j-1}(x_0) - D_{2j-1}\left(\frac{1}{8\text{Fo}}\right)\right] +$$

$$+ \frac{1}{128\text{Fo}_s(j+1)^2}\left[D_{2j-1}(x_0) - D_{2j-1}\left(\frac{1}{8\text{Fo}}\right)\right];$$

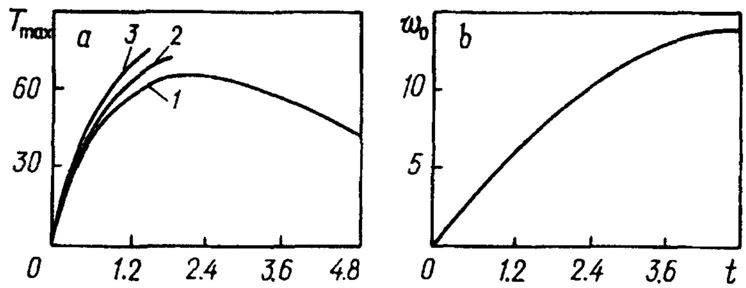

Fig. 1. Plot of temperature T_{max} in circular contact area (a) and of thermal distortion W_0 of the surface of brake disk (b) versus time: T_{max}, °C; w_0, μm; t, sec.

$$d_1(x) = -\frac{1}{x}\exp(-x) - E_1(x); \quad d_2(x) = \frac{1}{2x}\exp(-x)\left(1 - \frac{1}{x}\right) + \frac{3}{4}E_1(x);$$

$$D_m(\xi) = \exp(-\xi)\sum_{l=0}^{m}\frac{m!}{(m-l)!}\xi^{m-l}; \quad E_1(\zeta) = -\operatorname{Ei}(-\zeta) = \int_{\zeta}^{\infty}\frac{\exp(-\xi)}{\xi}d\xi$$

is an integral exponential function.

3. Let us study the time behavior of the temperature $T_{max}(t)$ and thermal distortion $W_0(t)$ of the surface of a disk made of cast iron with the following physical and mechanical characteristics: $\nu = 0.26$; $k = 1.286 \cdot 10^{-5}$ m^2/sec; $K = 50$ W/m·°C; $\alpha_t = 12 \cdot 10^{-6}$ °C^{-1}. The area of the lateral surface of the disk $A_a = 329 \cdot 10^{-4}$ m^2 (the nominal contact area); the area of the working surface of the block $A_w = 32.9 \cdot 10^{-4}$ m^2, i.e., the mutual overlap coefficient is 0.1; the braking time $t_s = 4.8$ sec.

In Fig. 1a one can see curves of temperature versus time (curve 1 corresponds to the temperature determined from formula (8); curves 2 and 3 correspond to formulas (9) and (10), respectively), and Fig. 1b illustrates the time behavior of the thermal distortion of the contact area calculated from formulas (18) and (19). The numerical values presented show that relation (9) gives good results at $t < 0.72$ sec, and (10) is good at $t < 0.48$ sec. As can be seen from the plots presented, the temperature of the disk surface is maximal at $t = 2.4$ sec, while the thermal displacement increases during the entire braking period.

The present calculations are compared with the known experimental data of [11], where it was found by dynamometry that in deceleration of an automobile with a weight of one ton from 96.6 km/h to a full stop, the load P per block was, on average, about 680 N. The initial temperature of the disk surface measured by a thermocouple at the start of braking was 175°C, and the maximum temperature in the process of braking was 215°C, i.e., the temperature burst was 40°C. The amount of friction heat released in the braking process was $Q_* = 119.7 \cdot 10^3$ J. Then, the initial heat flux in the disk

$$q_0 = \frac{Q_*}{A_a t_s} = 757.9 \cdot 10^9 \quad \text{W/m}^2,$$

and it follows from (8) that the temperature of the surface of the disk at the end of braking was 45°C, which agrees well with the experimental 40°C.

A study of wear products produced by friction contact of a disk and blocks carried out in [12] showed that they were formed at 475°C, which exceeds greatly the calculated temperature indicated above. Therefore, the method of determining the heating temperature of the disk needs some refinements. In particular, it should be taken into consideration that in the course of braking, wear takes place only at the contact of the block with the disk, i.e., in the actual contact area

$$A_r = \frac{P}{\text{HM}}. \tag{20}$$

In this case the time of contact of the surface of the disk with the block is defined by the following relation

$$t = \frac{t_s A_r}{A_w}. \tag{21}$$

For the material considered (alloy cast iron), Meyer's hardness decreases rapidly as the temperature increases, and at $T = 475°C$ it is equal to 40 MPa [13]. Then it follows from (20) that $A_r = 17 \cdot 10^{-5}$ m^2. Substituting this value of the actual contact area into (21) gives the contact time $t = 0.248$ sec. In this case, the initial heat flux density $q_0 = 7579 \cdot 10^3$ W/m^2, and from Eqs. (8) and (9), $T = 298°C$ and $T = 303°C$, respectively, which is in good agreement with the experimental temperature "burst" of 300°C.

NOTATION

A_a, nominal contact area; A_r, actual contact area; A_w, working surface area of block; c_v, heat capacity per unit volume; k, thermal diffusivity; erf ζ, probability integral; K, thermal conductivity; $T(r, t)$, temperature field; r, radial coordinate; t, time; t_s, braking time; Fo$_s$, dimensionless braking time; q_0, friction heat flux density at initial moment; ν, Poisson's ratio; α_t, temperature coefficient of linear expansion; $H(\cdot)$, Heaviside unit function; $I_0(\cdot)$, modified first-order Bessel functions; HM, Meyer's hardness.

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